Appendix

In this appendix, I explicate a formal theoretic model of delegation and oversight and use it to generate predictions about how institutional variables affect levels of ex ante delegation of discretion and ex post oversight as legislative policymaking strategies. The text of the article describes the insights I have taken from this model in an informal way, but I have included this appendix so that interested readers may see the more formal results.

Assumptions

Assumptions are stated in the text of the paper. I denote the "Legislator" here as L and the "Bureaucrat" as B. L has an ideal point at $x_L = 0$, and B has an ideal point at some $x_B \geq 0$.

Sequence

Stage1

Nature determines a policy shock, $\varepsilon \in \{0,1\}$. The outcome of any policy, y, is $y - \varepsilon$. B knows the value of ε, but L believes that $\varepsilon = 1$ with probability p and that $\varepsilon = 0$ with probability $1-p$. Since this is a signaling game, L uses Bayes' rule to update these beliefs based on the B's actions.

Stage 2

L chooses to adopt some law $x \in \{0, 1, \bar{I}\}$, where $\bar{I}$ is the maximal upper bound to discretion, and $\bar{I} = x_B + 1$.[1] L must pay a cost k for limiting discretion, where $k = \left(a - \frac{ax}{\bar{I}}\right)$, so $x = \bar{I} \Rightarrow k = 0$ and $x = 0 \Rightarrow k = a$. Assume $a < \bar{I}$. Here, a is the legislative capacity variable. As it decreases, L is able to write more restrictive laws with less cost. Therefore, a certain threshold of capacity is needed to write a moderately detailed law ($x = 1$), but more capacity is always required to write the most detailed law ($x = 0$).

Stage 3

B implements a policy $\in \{0, 1, a, a+1, x_B, x_B+1\}$, called y_1 if $\varepsilon = 1$ and y_0 if $\varepsilon = 0$. [2] The policy B implements may be legal (i.e., $y_\varepsilon < x$) or illegal (i.e., $y_\varepsilon \geq x$). In Huber and Shipan, the outcome is determined by what B implements (minus ε) and exogenous nonstatutory factors included below in stage 4. The most important difference between my model and Huber and Shipan's is that there is a fourth stage where L has an opportunity to learn about the value of ε based on B's actions and to use this information to her advantage.

Stage 4

L observes the policy implemented by B and can choose to investigate or not. If she investigates, then the outcome goes to L's ideal point (i.e., any y becomes ε). If L does not investigate, then the outcome is $y_\varepsilon - \varepsilon$. The cost of investigating is given by a. If L investigates and B has acted outside

[1] L is limited to choosing among a maximal discretion law ($x = \bar{I}$, a law giving no specific instructions to B), a minimal discretion law ($x = 0$, a law giving comprehensive instructions to B), or something in between ($x = 1$). This discrete action space captures the general idea that legislatures can delegate across a continuum of discretion, but would only write statutes that are dominant over potential laws other than these archetypes. Huber and Shipan (2002) show, by elimination of strictly dominated strategies, that $x = 0$ and $x = 1$ are the only possible low discretion laws.

[2] As for L above, this discrete action space represents the dominant implementation decisions for B. Policy implemented at 0, 1, or $x_B + 1$ corresponds to B implementing the exact policy mandated by L. I let B potentially act illegally or to use his informational advantage by allowing for implementation decisions at his ideal point (x_B) and at indifference thresholds for L's subsequent investigation choice (a and $a+1$).

the bounds of discretion, that is, illegally, B must pay $d > 0$.[3] With some exogenous probability γ, the outcome reverts to L's ideal point and B pays d if he has implemented an illegal policy. This parameter represents nonstatutory, nonoversight mechanisms that may benefit L, such as the courts, the presence of a legislative veto, or the influence of interest groups over policy outcomes.

Solution Concept

Since this is a signaling game, the equilibria presented below are perfect Bayesian, which requires that players' beliefs be sequentially rational and determined by Bayes' rule when possible. My strategy for characterizing the equilibria is to do so in terms of B's position relative to L and other parameters of the model. I begin by characterizing the separating equilibria, then give conditions for the existence and character of semi-separating equilibria and show that the substantively interesting set of pooling strategies are unsustainable for this model. I then portray comparative statics.

Separating Equilibria

Where $x_B < a$, Figure 1

B's Strategy

Assume for now that L writes $x = \bar{I}$. In a separating PBE, each type of B chooses a different message, so that L may perfectly infer B's type given the policy they implement. Here, B can receive his ideal point x_B as an outcome by implementing $y_0 = x_B$ for type 0 or $y_1 = x_B + 1$ for type 1. Let us assume that this is B's strategy:

$$\sigma_B(t) = \begin{cases} x_B & \text{if } t = t_0 \\ x_B + 1 & \text{if } t = t_1 \end{cases}$$

L's Beliefs

Let $\mu(t_i|y_\varepsilon)$ be the probability that L assigns to type i after observing B's action y_B. When L observes B implement x_B, she will assign probability 1 to B being type 0. Likewise, when L observes B implement $x_B + 1$, she will assign probability 1 to B being of type 1. To illustrate this, consider Bayes' rule:

$$\mu(t_0|x_B) = \frac{P(x_B|t_0)P(t_0)}{P(x_B)} = \frac{P(x_B|t_0)P(t_0)}{P(x_B|t_0)P(t_0) + P(x_B|t_1)P(t_1)}$$

$P(t_0) = 1 - p$, $P(x_B|t_0) = 1$, and $P(x_B|t_1) = 0$, so when we substitute these probabilities into Bayes' rule, we see that the only belief consistent with it is for $\mu(t_0|x_B) = 1$. Similarly, $\mu(t_0|x_B + 1) = 0$, $\mu(t_1|x_B + 1) = 1$, and $\mu(t_1|x_B) = 0$.

L's Best Response

L considers her best response by comparing the expected utilities associated with Investigating and Not Investigating.

Against $y_\varepsilon = x_B$:

$$EU_L(I, x_B) = \mu(t_0|x_B) * U_L(I, x_B; t_0) + \mu(t_1|x_B) * U_L(I, x_B; t_1) = -a$$

and

$$EU_L(NI, x_B) = \mu(t_0|x_B) * U_L(NI, x_B; t_0) + \mu(t_1|x_B) * U_L(NI, x_B; t_1) = -x_B(1 - \gamma)$$

[3] This represents both the opportunity costs associated with the agency's time and staff resources needed to prepare for an oversight hearing and the potential political costs associated with being publicly embarrassed by being brought in front of a committee.

Since we know that $x_B < a$ in this region and that γ is a probability between 0 and 1, L's best response to $y_i = x_B$ is to Not Investigate.
Against $y_\varepsilon = x_B + 1$*:*

$$EU_L(I, x_B + 1) = \mu(t_0|x_B + 1) * U_L(I, x_B + 1; t_0) + \mu(t_1|x_B + 1) * U_L(I, x_B + 1; t_1) = -a$$

and

$$EU_L(NI, x_B + 1) = \mu(t_0|x_B + 1) * U_L(NI, x_B + 1; t_0) + \mu(t_1|x_B + 1) * U_L(NI, x_B + 1; t_1) =$$

$$-x_B(1 - \gamma)$$

Again, since $x_B < a$ in this region, L's best response to $y_i = x_B + 1$ is to Not Investigate.

Equilibrium
Since L's beliefs are Bayesian by construction and her strategy is a best response given those beliefs, this is an equilibrium only if B has no incentive to deviate. Given L's strategy, either type of B receives the highest possible utility by implementing $y_o = x_B$ and $y_1 = x_B + 1$, respectively, so there is never an incentive for them to deviate from this strategy.[4] Knowing this, L would never wish to investigate either on or off the equilibrium path. Finally, given this subgame, L would not deviate from writing the costless law ($x = \bar{I}$), since such deviation would not change B's strategy (since B's strategy here is dominant) and would only take away from L's utility.
The following is a separating PBE where we would expect L to neither limit discretion ex ante nor conduct ex post investigations:

$$\sigma_B(t) = \begin{cases} x_B & \text{if } t = t_0 \\ x_B + 1 & \text{if } t = t_1 \end{cases}$$

$$\sigma_L(y_i, \mu(y_\varepsilon)) = \begin{cases} x = \bar{I}, NI & \text{if } y_\varepsilon = x_B \\ x = \bar{I}, NI & \text{if } y_\varepsilon = x_B + 1 \end{cases}$$

$$\mu(y_\varepsilon) = \begin{matrix} (\mu(t_0|x_B) = 1 \\ (\mu(t_0|x_B + 1) = 0 \end{matrix}$$

Proposition 1. *When $x_B < a$, L never limits discretion ex ante nor conducts ex post investigations.*

Where $a \leq x_B \leq a + \frac{1}{2}$, Figure 2
B's Strategy
Assume for now that L writes $x = \bar{I}$. B can no longer receive his ideal point through implementation because L would prefer to pay the cost a to investigate and force the outcome to 0. B would lose less policy utility by choosing to implement a policy which yields an outcome at a. B would always prefer

[4] Generally, to show that an equilibrium is a PBE, one must set arbitrary beliefs (as I do for L in the pooling case below) for the signal receiver at information sets that are not reached along the equilibrium path. These off the path beliefs need not be determined by Bayes' rule and any beliefs can support a PBE as long as they would not make the sender (B in this model) wish to deviate from their equilibrium strategy. In this case, B's equilibrium strategy is dominant (it yields his ideal point for any of L's beliefs), so actually assigning L's off the path beliefs would be superfluous here. B sending the signal assigned on the equilibrium path is optimal for him.

this outcome to 0 and would avoid paying the cost of being investigated, d. Therefore, let us assume that B implements $y_0 = a$ for type 0 or $y_1 = a+1$ for type 1.

$$\sigma_B(t) = \begin{cases} a & \text{if } t = t_0 \\ a+1 & \text{if } t = t_1 \end{cases}$$

L's Beliefs
Again constructing L's beliefs via Bayes' rule, we find that $\mu(t_0|a) = 1$, $\mu(t_0|a+1) = 0$, $\mu(t_1|a+1) = 1$, and $\mu(t_1|a) = 0$.

L's Best Response
Against $y_\varepsilon = a$*:*

$$EU_L(I,a) = \mu(t_0|a) * U_L(I,a;t_0) + \mu(t_1|a) * U_L(I,a;t_1) = -a$$

and

$$EU_L(NI,a) = \mu(t_0|a) * U_L(NI,a;t_0) + \mu(t_1|a) * U_L(NI,a;t_1) = -a(1-\gamma)$$

Since γ is a probability, it is between 0 and 1, and for all values besides 0, L would strictly prefer to Not Investigate here.[5]
Against $y_\varepsilon = a+1$*:*

$$EU_L(I,a+1) = \mu(t_0|a+1) * U_L(I,a+1;t_0) + \mu(t_1|a+1) * U_L(I,a+1;t_1) = -a$$

and

$$EU_L(NI,a+1) = \mu(t_0|a+1) * U_L(NI,a+1;t_0) + \mu(t_1|a+1) * U_L(NI,a+1;t_1) = -a(1-\gamma)$$

Similarly, L would prefer to Not Investigate when she sees B implement $a+1$ in this preference arrangement.

Equilibrium
Since L's beliefs are Bayesian by construction and her strategy is a best response given those beliefs, this is an equilibrium only if B has no incentive to deviate. B's utility from not deviating, regardless of type, is $-(x_B - a)$. If B deviated, it would change L's beliefs about B's type and would therefore change the expected utilities for L such that she would prefer to investigate all the time, thereby changing B's expected utility. Here, the outcome would yield a utility for either type of B of $-x_B - d$. Since $x_B > a$ in this region, this utility would always be strictly lower than $-(x_B - a)$. Therefore, B will not deviate from the given strategy in equilibrium. Likewise, L would not limit discretion by writing $x < \bar{I}$ because it would strictly add cost to her utility. Since neither player would deviate from their strategies, the following is a PBE:

$$\sigma_B(t) = \begin{cases} a & \text{if } t = t_0 \\ a+1 & \text{if } t = t_1 \end{cases}$$

$$\sigma_L(y_i, \mu(y_\varepsilon)) = \begin{cases} x = \bar{I}, NI & \text{if } y_\varepsilon = a \\ x = \bar{I}, NI & \text{if } y_\varepsilon = a+1 \end{cases}$$

$$\mu(y_\varepsilon) = \begin{matrix} (\mu(t_0|a) = 1 \\ (\mu(t_0|a+1) = 0 \end{matrix}$$

[5]For $\gamma = 0$ I will assume, given the indifference between Investigating and Not Investigating, that L would prefer to Not Investigate since it would require taking additional action to achieve the same utility.

Proposition 2. *When $a \leq x_B \leq a+\frac{1}{2}$, L never limits discretion ex ante nor conducts ex post investigations.*

And, combining the first two propositions,

Corollary 1. *When $x_B \leq a+\frac{1}{2}$, L never limits discretion ex ante nor conducts ex post investigations.*

Pooling and Semi-Separating Equilibria

Pooling

There are no separating equilibria when x_B becomes too large ($x_B > a+\frac{1}{2}$, Figure 3). To see this, assume that when $\varepsilon = 1$, B implements $y_1 = a+1$, yielding an outcome, as before, at a. Now consider the case where $\varepsilon = 0$. B would prefer an outcome at $a+1$ to one at a, so he has an incentive to "cheat" here and again implement $a+1$. This is considered a "pooling" strategy, which is characterized by both types of B behaving in the same way. If there exists a pooling PBE here, both types of B must implement $y_B = a+1$.

$$\sigma_B(t) = \begin{cases} a+1 & \text{if } t = t_0 \\ a+1 & \text{if } t = t_1 \end{cases}$$

When L sees B implement $a+1$ here, she uses Bayes' rule to update her beliefs about type.

$$\mu(t_0|a+1) = \frac{P(a+1|t_0)P(t_0)}{P(a+1)} = \frac{P(a+1|t_0)P(t_0)}{P(a+1|t_0)P(t_0)+P(a+1|t_1)P(t_1)}$$

Since, by assumption, $P(a+1|t_0) = 1$ and $P(a+1|t_1) = 1$, and, by construction, $P(t_0) = 1-p$ and $P(t_1) = p$, we get $\mu(t_0|a+1) = 1-p$. Similarly, by Bayes' rule, $\mu(t_1|a+1) = p$. On the pooling equilibrium path, these posterior beliefs are exactly the same as the prior probabilities of being in each state of the world. L does not learn anything from B's behavior when the two types of B pool. If Bureaucrat t_0 were to implement a, off the equilibrium path, Bayes' rule would not apply: $\mu(t_0|a) \neq \frac{P(a|t_0)(P(t_0)}{P(a)}$, because $P(a) = 0$. Therefore, to check off-path beliefs, we need to arbitrarily assign them to see if they can support a pooling equilibrium. I assume that $\mu(t_0|a) = \lambda \in [0,1]$.

The Legislator's best response, since $EU_L(I,a+1) > EU_L(NI,a+1)$, to on-the-path play is to Investigate B with probability 1 when $a > 1$:

$$EU_L(I,a+1) = \mu(t_0|a+1) * U_L(I,a+1;t_0) + \mu(t_1|a+1) * U_L(I,a+1;t_1) = -a$$

$$EU_L(NI,a+1) = \mu(t_0|a+1) * U_L(NI,a+1;t_0) + \mu(t_1|a+1) * U_L(NI,a+1;t_1) =$$
$$-pa + (1-p)(-|a-1|)$$

Likewise, L will Investigate off-the-path behavior with $\mu(t_0|a) = \lambda$ and $\mu(t_0|a) = 1-\lambda$ only when $a > 1$, otherwise she would prefer to Not Investigate:

$$EU_L(I,a) = \mu(t_0|a) * U_L(I,a;t_0) + \mu(t_1|a) * U_L(I,a;t_1) = -a$$

$$EU_L(NI,a) = \mu(t_0|a) * U_L(NI,a;t_0) + \mu(t_1|a) * U_L(NI,a;t_1) = -\lambda a + (1-\lambda)(-|a-1|)$$

$$BR_L(a|\mu(t_0|a)) = \begin{cases} I & \text{if } a > 1 \\ NI & \text{if } a \leq 1 \end{cases}$$

Either type of B would prefer to deviate from their strategy profile and play the strategy that would yield their ideal points (i.e., $y_0 = x_B$ and $y_1 = x_B + 1$) when $a \leq 1$, given L's off-the-equilibrium path beliefs. Therefore, the pooling strategies assigned to B do not support equilibrium in this model.

Semi-separating ($x_B > a + \frac{1}{2}$), Figure 3

Maximal Discretion law ($x = \bar{I}$)

Let us now consider potential equilibria with type t_1 implementing $a+1$, as in the above consideration of pooling strategies, but let us assume that type t_0 "cheats" (i.e., implements $y_0 = a+1$) with probability q and does not cheat (i.e., implements $y_0 = a$) with $1-q$. We here consider the case where t_1 plays a pure strategy and t_0 mixes because the preference arrangement implies that t_0 has the opportunity to use his informational advantage to gain policy from the imperfectly updating L.

B's Strategy

In general,

$$\sigma_B(t) = \begin{cases} a & \text{with probability } 1-q \quad \text{if } t = t_0 \quad q \in (0,1] \\ a+1 & \text{with probability } q \quad \text{if } t = t_0 \\ a+1 & \text{if } t = t_1 \end{cases}$$

L's Beliefs

Since both $a+1$ and a are played on the equilibrium path, L's beliefs follow Bayes' rule for each information set. According to B's strategy, we know that $P(a+1|t_1) = 1$ and $P(a+1|t_0) = q$, and by construction $P(t_1) = p$ and $P(t_0) = 1-p$, so we get:

$$\begin{aligned} \mu(t_1|a) &= 0 \\ \mu(t_1|a+1) &= \tfrac{P(a+1|t_1)P(t_1)}{P(a+1)} = \tfrac{P(a+1|t_1)P(t_1)}{P(a+1|t_1)P(t_1)+P(a+1|t_0)P(t_0)} = \tfrac{p}{p+q-pq} \end{aligned}$$

B's Equilibrium Strategy

In equilibrium, a B of type t_0 must choose the probability q with which he "cheats" and implements $a+1$ instead of a. This probability needs to be chosen so as to make L indifferent about Investigating him (i.e., make $EU_L(I|a+1) = EU_L(NI|a+1)$). If L is not indifferent about Investigating, a B of type t_0 would no longer be willing to mix strategies and the semi-separating equilibrium would not be supported. As with the separating cases, these expected utilities are determined by L's beliefs about B's type and the objective utilities associated with the potential outcomes. Where $\mu = \mu(t_1|a+1) = \frac{p}{p+q-pq}$ and $1-\mu = (t_0|a+1) = 1 - \frac{p}{p+q-pq}$:

$$EU_L(I|a+1) = EU_L(NI|a+1)$$

$$(1-\mu) * U_L(I, a+1; t_0) + \mu * U_L(I, a+1; t_1) = (1-\mu) * U_L(NI, a+1; t_0) + \mu * U_L(NI, a+1; t_1)$$

$$-a = (1-\mu)(1-\gamma)(-(a+1)) + (\mu)(1-\gamma)(-a)$$

Substituting $\frac{p}{p+q-pq}$ for μ:

$$-a = -\frac{p(1-\gamma)a}{p+q-pq} - \left(1 - \frac{p}{p+q-pq}\right)(1-\gamma)(a+1)$$

And solving for q:

$$q = \frac{ap\gamma}{1 - \gamma a - \gamma - p + ap\gamma + p\gamma}$$

L's Best Response

Since she is indifferent between the two, L will respond to B's strategy by mixing over Investigating and Not Investigating. In equilibrium, L will choose a probability of Investigation i which makes the B of type t_0 indifferent about cheating, so that he would not prefer to cheat all of the time.

$$EU_B(a|t_0) = EU_B(a+1|t_0)$$

$$\gamma(-x_B) + (1-\gamma)(-(x_B-a)) = i(-x_B) + (1-i)[\gamma(-x_B) + (1-\gamma)(-a-1+x_B)]$$

After simplifying, and solving for i, we get:

$$i = \frac{2a+1-2x_B}{a+1-2x_B}$$

Now that we have these equilibrium mixing probabilities, we can construct utilities for L given the types of B, and eventually expected utilities given B's strategies.

$$\begin{aligned} U_L(a+1|t_0) &= -ia - (1-i)(1-\gamma)(a+1) \\ &= -\tfrac{(2a+1-2x_B)a}{a+1-2x_B} - \left(1 - \tfrac{2a+1-2x_B}{a+1-2x_B}\right)(1-\gamma)(a+1) \end{aligned}$$

$$\begin{aligned} U_L(a|t_0) &= (1-\gamma)(-a) \\ &= -(1-\gamma)a \end{aligned}$$

$$\begin{aligned} U_L(a+1|t_1) &= -ia - (1-i)(1-\gamma)(a) \\ &= -\tfrac{(2a+1-2x_B)a}{a+1-2x_B} - \left(1 - \tfrac{2a+1-2x_B}{a+1-2x_B}\right)(1-\gamma)a \end{aligned}$$

We substitute these utilities into L's expected utility for this maximal discretion subgame, and after some simplification, we get:

$$\begin{aligned} EU_L(x=\bar{I}) &= (1-p)[q*U_L(a+1|t_0) + (1-q)*U_L(a|t_0)] + p*U_L(a+1|t_1) \\ &= \tfrac{a\left(\gamma^2 a - \gamma a - p\gamma^2 + \gamma^2 + 1 + p\gamma - 2\gamma\right)}{-1+\gamma a+\gamma} \end{aligned}$$

L's beliefs are Bayesian by construction, her strategy is a best response given these beliefs, and B's strategy is constructed as a best response to L's strategy, so this is a semi-separating equilibrium if L has no incentive to deviate from writing a costless law ($x = \bar{I}$). In the next section, I show that L has no such incentive for many values of γ, but will limit B's discretion by writing $x = 1$ for other values of γ.

Limited Discretion law ($x = 1$)
In the subgame where L writes $x = 1$ instead of $x = \bar{I}$, she incurs a constant cost of $k = a - \frac{a}{x_B+1}$. In addition, B must now pay a cost d when he is caught cheating (in this case, this means implementing $a+1 > 1$). These changes affect the structure of the i parameter and the critical utilities of L.

B's Strategy

$$\sigma_B(t) = \begin{cases} a & \text{with probability } 1-q \quad \text{if } t = t_0 \quad q \in (0,1] \\ a+1 & \text{with probability } q \quad \text{if } t = t_0 \\ a+1 & \text{if } t = t_1 \end{cases}$$

L's Beliefs

$$\mu(t_1|a) = 0$$
$$\mu(t_1|a+1) = \frac{P(a+1|t_1)P(t_1)}{P(a+1)} = \frac{P(a+1|t_1)P(t_1)}{P(a+1|t_1)P(t_1)+P(a+1|t_0)P(t_0)} = \frac{p}{p+q-pq}$$

B's Equilibrium Strategy

Since B's equilibrium strategy is chosen to make L indifferent about Investigating, the introduction of the possibility of B being punished for acting illegally does not change his general strategy:

$$q = \frac{ap\gamma}{1-\gamma a-\gamma-p+ap\gamma+p\gamma}$$

L's Best Response

The introduction of the d penalty for when the Bureaucrat is investigated when he acts illegally affects the probability that L holds a hearing. To see this, consider again that L chooses this probability so as to make B indifferent about cheating.

$$EU_B(a|t_0) = EU_B(a+1|t_0)$$

$$\gamma(-x_B)+(1-\gamma)(-(x_B-a)) = i(-x_B-d)+(1-i)[\gamma(-x_B-d)+(1-\gamma)(-a-1+x_B)]$$

After simplifying, and solving for i, we get:

$$\frac{-2x_B+2a+2\gamma a-2\gamma x_B+\gamma d+1+\gamma}{-2x_B-d+a+1+\gamma a+\gamma-2\gamma x_B+\gamma d}$$

As before, we can construct utilities and expected utilities for the limited discretion subgame with these mixing probabilities. Importantly, these utilities include the cost of writing the limited discretion law, $k = a-\frac{a}{x_B+1}$.

$$\begin{aligned} U_L(a+1|t_0) &= (-ia-(1-i)(1-\gamma)(a+1))-k \\ &= -\frac{(-2x_B+2a+2\gamma a-2\gamma x_B+\gamma d+1+\gamma)a}{-2x_B-d+a+1+\gamma a+\gamma-2\gamma x_B+\gamma d} - \left(1-\frac{-2x_B+2a+2\gamma a-2\gamma x_B+\gamma d+1+\gamma}{-2x_B-d+a+1+\gamma a+\gamma-2\gamma x_B+\gamma d}\right) \\ &\quad *(1-\gamma)(a+1)-a+\frac{a}{x_B+1} \end{aligned}$$

$$\begin{aligned} U_L(a|t_0) &= ((1-\gamma)(-a))-k \\ &= -(1-\gamma)a-a+\frac{a}{x_B+1} \end{aligned}$$

$$\begin{aligned} U_L(a+1|t_1) &= (-ia-(1-i)(1-\gamma)(a))-k \\ &\quad -\frac{(-2x_B+2a+2\gamma a-2\gamma x_B+\gamma d+1+\gamma)a}{-2x_B-d+a+1+\gamma a+\gamma-2\gamma x_B+\gamma d} - \left(1-\frac{-2x_B+2a+2\gamma a-2\gamma x_B+\gamma d+1+\gamma}{-2x_B-d+a+1+\gamma a+\gamma-2\gamma x_B+\gamma d}\right) \\ &\quad *(1-\gamma)a-a+\frac{a}{x_B+1} \end{aligned}$$

L's expected utility for the limited discretion subgame:

$$\begin{aligned} EU_L(x=\bar{I}) &= (1-p)[q*U_L(a+1|t_0)+(1-q)*U_L(a|t_0)]+p*U_L(a+1|t_1) \\ &= \frac{a\left(\gamma^2 ax_B+\gamma^2 a-2\gamma ax_B-a-\gamma a-p\gamma^2+\gamma^2 x_B-p\gamma^2 x_B+\gamma^2-3\gamma x_B+p\gamma+p\gamma x_B-2\gamma+2x_B+1\right)}{(x_B+1)(-1+\gamma a+\gamma)} \end{aligned}$$

L's beliefs are Bayesian by construction, her strategy is a best response given these beliefs, and B's strategy is constructed as a best response to L's strategy, so this is a semi-separating equilibrium if L has no incentive to deviate from writing a limited discretion law ($x = 1$).

To see when L would write the limited discretion law instead of the maximal discretion one, we must compare the expected utilities of each. By assuming that the $EU_L(x=\bar{I}) > EU_L(x=1)$, and then solving the inequality for the exogenous γ, we see that there is a middle region of γ where L would prefer to write the limited discretion over the costless maximal discretion law.
For $\gamma > \frac{1}{a+1}$ (which makes $-1+\gamma a+\gamma$ positive):

$$\begin{aligned} EU_L(x=\bar{I}) &> EU_L(x=1) \\ \frac{a\left(\gamma^2 a-\gamma a-p\gamma^2+\gamma^2+1+p\gamma-2\gamma\right)}{-1+\gamma a+\gamma} &> \\ \frac{a\left(\gamma^2 a x_B+\gamma^2 a-2\gamma a x_B-a-\gamma a-p\gamma^2+\gamma^2 x_B-p\gamma^2 x_B+\gamma^2-3\gamma x_B+p\gamma+p\gamma x_B-2\gamma+2x_B+1\right)}{(x_B+1)(-1+\gamma a+\gamma)} & \end{aligned}$$

$$\gamma^2 a x_B+\gamma^2 a-\gamma a+p\gamma x_B+p\gamma-p\gamma^2 x_B-p\gamma^2+x_B+1+\gamma^2 x_B+\gamma^2-2\gamma x_B-2\gamma >$$
$$\gamma^2 a x_B+\gamma^2 a-2\gamma a x_B-a-\gamma a-p\gamma^2+\gamma^2 x_B-p\gamma^2 x_B+\gamma^2-3\gamma x_B+p\gamma+p\gamma x_B-2\gamma+2x_B+1$$

Solving for γ:

$$\gamma > -\frac{-x_B+a}{x_B(a+1)}$$

And for $\gamma < \frac{1}{a+1}$ (which makes $-1+\gamma a+\gamma$ negative):

$$\begin{aligned} EU_L(x=\bar{I}) &> EU_L(x=1) \\ \frac{a\left(\gamma^2 a-\gamma a-p\gamma^2+\gamma^2+1+p\gamma-2\gamma\right)}{-1+\gamma a+\gamma} &> \\ \frac{a\left(\gamma^2 a x_B+\gamma^2 a-2\gamma a x_B-a-\gamma a-p\gamma^2+\gamma^2 x_B-p\gamma^2 x_B+\gamma^2-3\gamma x_B+p\gamma+p\gamma x_B-2\gamma+2x_B+1\right)}{(x_B+1)(-1+\gamma a+\gamma)} & \end{aligned}$$

$$\gamma^2 a x_B+\gamma^2 a-\gamma a+p\gamma x_B+p\gamma-p\gamma^2 x_B-p\gamma^2+x_B+1+\gamma^2 x_B+\gamma^2-2\gamma x_B-2\gamma <$$
$$\gamma^2 a x_B+\gamma^2 a-2\gamma a x_B-a-\gamma a-p\gamma^2+\gamma^2 x_B-p\gamma^2 x_B+\gamma^2-3\gamma x_B+p\gamma+p\gamma x_B-2\gamma+2x_B+1$$

Solving for γ:

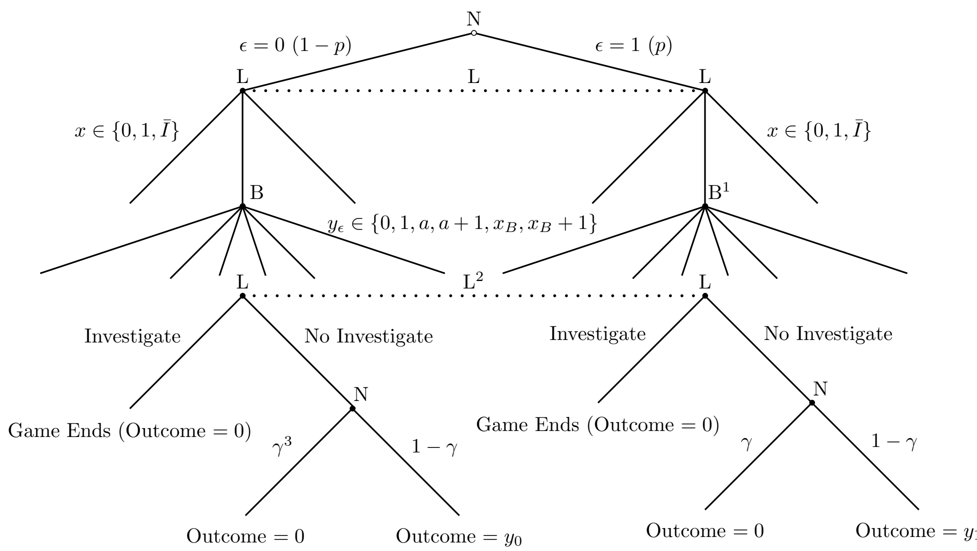
$$\gamma < -\frac{-x_B+a}{x_B(a+1)}$$

When γ is between these two values ($-\frac{-x_B+a}{x_B(a+1)} < \gamma < \frac{1}{a+1}$), $EU_L(x=\bar{I}) < EU_L(x=1)$ and L would prefer to limit discretion in equilibrium. However, when $\gamma \le -\frac{-x_B+a}{x_B(a+1)}$ or $\gamma > \frac{1}{a+1}$, then $EU_L(x=\bar{I}) > EU_L(x=1)$ and L would write the maximal discretion law in equilibrium.

Proposition 3. *When $x_B > a+\frac{1}{2}$ and γ is either sufficiently low ($\gamma < -\frac{-x_B+a}{x_B(a+1)}$) or sufficiently high ($\gamma > \frac{1}{a+1}$), L does not limit discretion ex ante, but does conduct ex post investigations with a probability, $i = \frac{2a+1-2x_B}{a+1-2x_B}$, that increases in x_B and decreases in a.*

Proposition 4. *When $x_B > a+\frac{1}{2}$ and γ is neither sufficiently low nor sufficiently high ($-\frac{-x_B+a}{x_B(a+1)} < \gamma < \frac{1}{a+1}$), L limits discretion ex ante and conducts ex post investigations with a probability, $i = \frac{-2x_B+2a+2\gamma a-2\gamma x_B+\gamma d+1+\gamma}{-2x_B-d+a+1+\gamma a+\gamma-2\gamma x_B+\gamma d}$, that increases in x_B and decreases in a.*

Figure 1: Simplified Representation of the Extensive Form



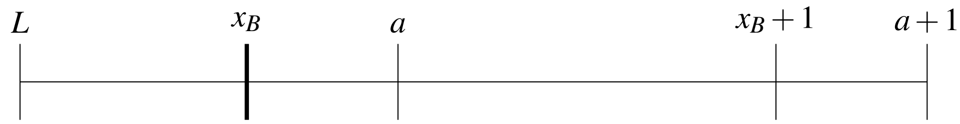
Notes:

1. At the stage where B implements L's law (stage 3), B has the same available actions for each law that L chooses in the previous stage. Likewise, L's strategies in stage 4 are for any given implementation decision of B.

2. Although L does not know the value of ϵ at this information set, she does know what her previous action ($x \in \{0, 1, \bar{I}\}$) was.

3. γ is the probability with which Nature reverts the policy outcome to L's ideal point. This represents exogenous nonstatutory determinants of policy outcomes.

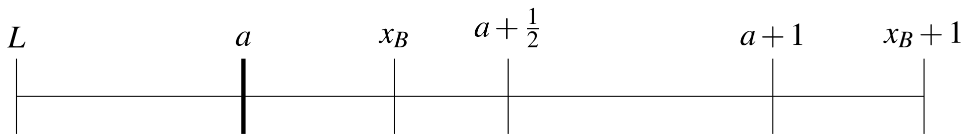
Figure 2: Region 1 — Separating Equilibrium



Separating Equilibrium: $x_B < a$

- L passes $x = x_B + 1$, B implements $y_1 = x_B + 1$ when $\varepsilon = 1$, L does not investigate
- L passes $x = x_B + 1$, B implements $y_0 = x_B$ when $\varepsilon = 0$, L does not investigate

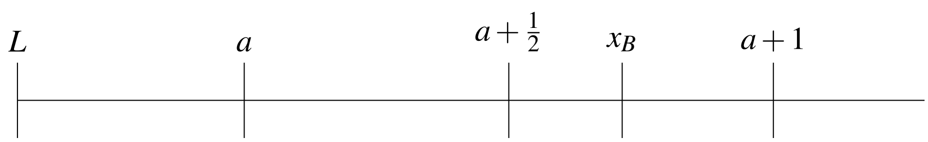
Figure 3: Region 2 — Separating Equilibrium



Separating Equilibrium: $a \leq x_B \leq a + \frac{1}{2}$

- L passes $x = x_B + 1$, B implements $y_1 = a + 1$ when $\varepsilon = 1$, L does not investigate
- L passes $x = x_B + 1$, B implements $y_0 = a$ when $\varepsilon = 0$, L does not investigate

Figure 4: Region 3 — Semi-separating Equilibrium



Semi-separating: $a + \frac{1}{2} < x_B$

- There is no pure separating strategy for B here
  - If $\varepsilon = 1$, B plays pure $a + 1$
  - If $\varepsilon = 0$, B mixes between $a + 1$ and a

Figure 5: Empirical Expectations when: $(-\frac{-x_B+a}{x_B(a+1)} < \gamma < \frac{1}{a+1})$